Anisotropic Correlation Properties in the Spatial Structure of Cortical Orientation Maps

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1 Introduction

Experimental and theoretical studies on visual information processing have evidenced the advantages of a delocalized representation of information through *feature maps* which mix topological contiguity with proximity in the feature space. The peculiar dispositions of cortical orientation-selective cells can be understood in terms of dimension-reducing mappings which translate neighborhood relations in the orientation-subspace in spatial neighborhood relations on the cortical surface [Durbin and Mitchison, 1990]. Thus, cortical maps are not only a repository of information about the features present in the image, but are the substrate for coordinated interactions among features. The basic computational principles of these maps can be investigated with different approaches (heuristic, theoretical information, computational). In this paper, we'll provide information-theoretic insights about spatial arrangements of simple cell receptive fields in several biologically plausible orientation maps, by studying both autocorrelation and directional mutual information of cortical activity.

2 Autocorrelation of cortical activity

The retino-cortical pathway can be modeled by two layers, interconnected through feed-forward connections. The layers represent the retina ($\mathbf{x}$) and the cortical surface ($\mathbf{u}$). If $i(\mathbf{x})$ is the visual input, the activity $e(\mathbf{u})$ of a cortical cell can be written as $e(\mathbf{u}) = \int\int w_{\mathbf{u}}(\mathbf{x})\, i(\mathbf{x})\, d\mathbf{x}$ where the linear transfer function $w_{\mathbf{u}}(\mathbf{x})$ represents the receptive field. In this paper, we assume that cells have the same elongated Gaussian receptive field profile (aspect ratio 3.0), indepentently of their position on the retinal plane, and that they are selective to a specific orientation θ that varies with cortical coordinates, according to a fixed orientation map. To estimate the capabilities of representation of visual information through a population of cells, we evaluate, in absence of noise, the spatial autocorrelation function of cortical activity $e(\mathbf{u})$. The autocorrelation function is strictly related to the statistics of input signal. However, to focus our attention only on the structural properties of orientation maps, we assumed that visual inputs are completely uncorrelated. Under this assumption, to evaluate the autocorrelation function of cortical activity it is sufficient to consider the deterministic autocorrelation function

[Papoulis, 1987] $Q(\mathbf{u};\mathbf{u}') = \int\int w_{\mathbf{u}}(\mathbf{x})\, w_{\mathbf{u}'}(\mathbf{x})\, d\mathbf{x}$. Given a cell located at position $\mathbf{u}^*$ on the cortical surface, the autocorrelation function centered around that cell is $Q(\mathbf{u}^*;\mathbf{u}^*+\mathbf{s})$. According to the characteristics of the orientation map, $Q(\mathbf{u}^*;\mathbf{u}^*+\mathbf{s})$ changes from point to point on the cortical surface. The values of the autocorrelation function can be used to define the ensemble of cells belonging to the hypercolumn centered around any given cell. In fact, $Q(\mathbf{u}^*;\mathbf{u}^*+\mathbf{s})$ is related to the maximum set of dependent channels (receptive fields) with respect to a reference cell, and it can be used to define the hypercolumn radius. In Fig. 1 three different maps generated through a model recently proposed by Wörgötter and Niebur [Wörgötter and Niebur, 1993] are shown. The shaded regions represent the hypercolumns referred to the cell located in the center of the map. The 2D profiles of the autocorrelation functions provide clues about the signal processing capabilities of the spatial arrangements of receptive fields within the hypercolumns, as well as about global properties of the whole map. The elongated shapes, observed in stripe-like maps as the one depicted in Fig. 1(left), point out that, in this type of map, there are preferential directions along which strongly correlated activities occur. It is worth noting that, from the point of view of coding theory, optimal mappings of input signals onto cortical surface, could be achieved, on the contrary, when autocorrelation functions are characterized by radial simmetry similar to the one observed for the more realistically-looking maps shown in Figs. 1(center) and 1(right).

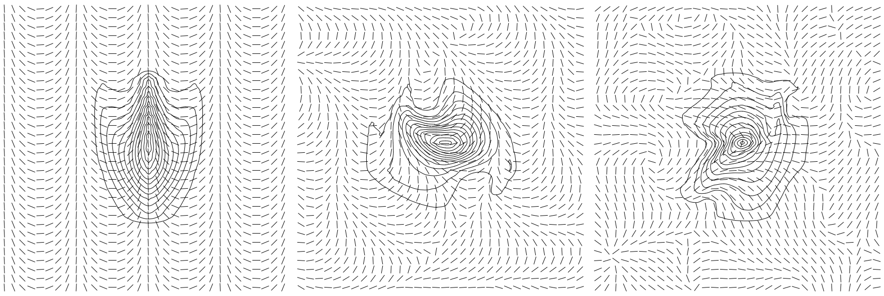

Figure 1: Contour plots of the autocorrelation function referred to the cell located in the center of the map. Their shapes depend on the background orientation maps evaluated according to [Wörgötter and Niebur, 1993].

3 Directional mutual information

More information about the anisotropies of information properties of hypercolumns can be obtained considering the mutual information of cortical activity and evaluating it along various direction departing from a given cell in the orientation map. The information capacity of an assembly of cells, that act as a single information-carrying channel, can be measured in terms of the conditional entropy of their activity. For any given direction ϕ departing from a fixed cortical cell, let's consider the conditional entropy $H_\phi\left(e_0 \mid e_1^\phi, \ldots, e_d^\phi\right)$, i.e. the uncertainty about the activity e_0 of a fixed cell, when the activity of d cells along the ϕ direction have been observed. For distances greater than

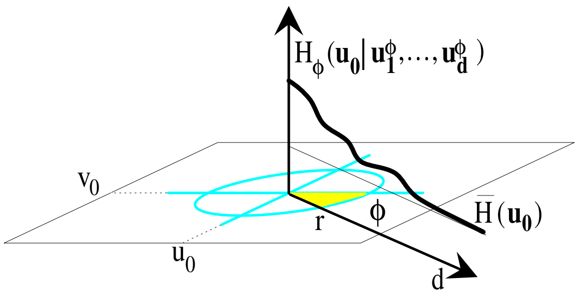

Figure 2: Illustration showing how directional entropy has been evaluated.

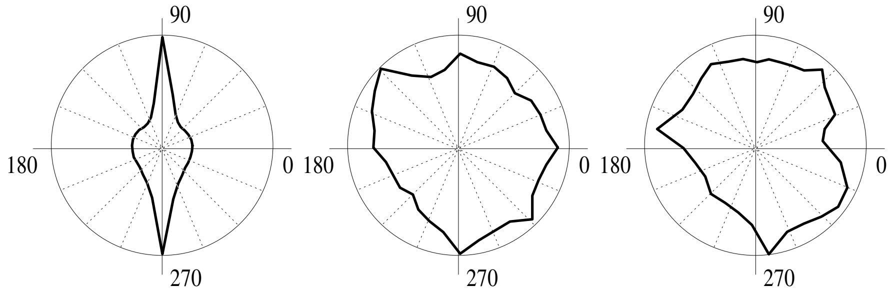

Figure 3: Polar plots of mutual information I_ϕ for the three hypercolumns shown in Fig. 1.

the hypercolumn radius the conditional entropy approaches its asymptotical value representing the *entropy rate* $\bar{H}_\phi(e_0)$ of the ensemble of cells along the direction ϕ(see Fig. 2). If the input signals are mutually spatially independent with normal distribution the entropy rate $\bar{H}_\phi(e_0)$ is related only to the spatial structure of the orientation map and the mutual information can be written as [Papoulis, 1987]

$$I_\phi\left(e_0, e_1^\phi, \ldots, e_\infty^\phi\right) = H(e_0) - \bar{H}_\phi(e_0) = \frac{1}{2}\left(\ln(S_0) - \lim_{d\to\infty} \ln \frac{\Delta_{d+1}^\phi}{\Delta_d^\phi}\right)$$

where Δ_d^ϕ is the determinant of the correlation matrix R_d^ϕ of the activity of the first d cells along the direction ϕ, and S_0 is the energy of the receptive field profile. I_ϕ measures the information that the activity of the ensemble of cells disposed along the direction ϕ conveys about the input signal.

The evaluation of directional mutual information along several direction and for different centers on the map evidences anisotropies in the amount of information. In Fig. 3 are shown the polar plots of mutual information I_ϕ; from left to right the diagrams refer to the maps depicted in Fig. 1. The stripe-like map (Fig. 3 left) reveals highest asymmetries in mutual information while patch-like maps achieve good simmetry in mutual information.

4 Discussion and Conclusion

Information processing capabilities of visual cortex can be evaluated at different levels of observation. At single cell level, one deals with the properties of receptive fields considered as local feature extractors. At global level the whole orientation map should be characterized to point out how these structured assemblies of neurons cooperate and gain higher levels of description (e.g. feature enhancement and texture segregation). This can be pursued investigating how information about any small locus in visual space affects the activity of nearby cells on the basis of their overlap and mutual interactions. In this paper, we have focussed our analysis on the overlap. The way in which neighboring receptive fields overlap depends on their shape and extent, and on their relative disposition dictated by the orientation map. This overlap affects the local representation of visual information and can be quantified considering the autocorrelation function of cortical activity. In order to have the same representation and computation capabilities across the visual space, it is necessary to have an equal allocation of cells of different orientation preference for each point (cf. the uniform *coverage* criterium of [Swindale, 1991]). We should prefer those orientation maps that exhibit average invariance characteristics of the autocorrelation functions, thus maximizing spatial uncertainty. Comparable quantitative conclusions can be drawn by measuring directional mutual information at each cortical locations, and using it as a further measure of uniformity of coverage. Under this perspective, the (an-)isotropies in both the autocorrelation function and in mutual information become figures of merit of orientation maps or act as constraints for the design of artificial orientation maps.

Using these quantitative criteria we can characterize different orientation maps. We have observed (see Fig. 1(center), 1(right) and also Fig. 3(center), 3(right)) that patch-like arrangements of orientations lead to isotropic autocorrelation functions, thus ensuring good space invariant properties. Conversely, strong anisotropies have been observed (see Fig. 1(left) and Fig. 3(left)) for stripe-like maps.

References

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