Coupled Diffusion Maps as Neural Computational Models for Texture Segmentation

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Abstract

The computational capabilities of 2D continuous distribution of locally-interacting neural cells are analyzed through nonlinear partial derivative equations of reaction-diffusion type. The resulting *coupled diffusion maps* can be interpreted as a phenomenological model for the processes that take place on visual cortical surface.

A point excitation on the map activates, in two antagonist populations, both local excitatory diffusion processes and spatially displaced inhibitory reactions through lateral clustered coupling. The nonlinear competition between the two populations leads to a macroscopic organized dynamic behavior that, under appropriate input conditions, results in oscillations. Superimposed excitatory and inhibitory couplings between localized portions of the orientation map impose a long-range coherence and the whole system evolves through different configurations of coherent states. Averaging the pattern of activity over a window comparable in size with those of a complete set of orientations, we gain instantaneous global perception supporting scene segmentation. Experiments on texture segmentation, conducted with artificial and natural textures, validates the approach.

1 Introduction

Biological approaches for image processing in computer vision have evidenced the advantages of a delocalized representation of information through *feature maps* which combine spatial contiguity with proximity in the feature space. By example, the peculiar dispositions of orientation-selective neurons in mammalian visual cortex [10] [2] can be seen as dimension-reducing mappings, that translate neighborhood relations in the orientation-subspace in spatial neighborhood relations on the cortical surface [6]. Thus, cortical maps are not only a repository of information about the features present in the image, but are the substrate on which to organize a locally-based set of excitatory and inhibitory interconnections among features with minimal communication overhead.

However, the spreading of features on the cortical surface poses, at some point, the problem of a global coding that could group processes among visual features that belong to a common object, and dissociate those features that belong to distinct objects, though they be equally close together in the cortical map. To avoid the combinatorial explosion of cardinal cells (i.e. those cells with highly specific response properties [1]), the one-to-one approach has now given the way to the much more powerful idea of cooperative coding in which the "grand-mother" is no longer a single cell, but the concerted activity of a population distributed over the cortical surface [8]. Since synchronous oscillations in different regions of visual cortex reflect global properties of the stimulus [7] [9], the temporal characteristics of cortical activity can be used to build a relational code that can provide a solution to the features' binding problem in distributed neural networks [12] [16] [19] [17]. After these motivations, a great interest is devoted to the study of the computational properties of cortical masses modeled as a continuum.

In this paper, we present a phenomenological model of perceptual processes occurring in visual cortex during segmentation. The dynamic of the system is based on reaction-diffusion equations. It is noteworthy that this description does not aim to explain or model neurophysiological data in detail, but rather to evidence possible strategies of perceptual organization that can lead to formal networks or hardware devices for neuronal information processing.

2 A macroscopic approach to cooperative neural activity

The evolution of the behavior of a neural mass, occurs through successive activations of neural assemblies that define a new state or a new set of states for the mass. Such states are often called *macrostates* and are characterized by widespread cooperative activity of neurons. Following the standard scheme of continuum neurodynamics we regard such neural mass as a continuous distribution of neurons and synapses. At this level of description, the time and space scales of macroscopic neural events are much more longer than the scales for the individual neural events that are considered to be inaccessible to observation. The macrostates of a cortical mass are thus coarse-grained expressions of local activities.

2.1 The reaction-diffusion computational paradigm

At macroscopic level, the modeling of neural activity is drastically simplified on the basis of the following assumptions: (i) all nervous processes of any complexity are dependent upon isolated pairs of interactive excitatory and inhibitory populations [20], hereafter referred to as *dipole cells*; (ii) the flow of information across the neural mass is mediated by diffusion processes that spread activation across the cortical surface. As well as pair interactions are fundamental in priming cooperative processes, the spreading of activation over the cortical surface plays a relevant role in long-distance transfer of information across the neural mass. These processes are analogous to the diffusion of reacting chemical species and can be modeled by the following *reaction-diffusion* (RD) equation:

$$\frac{\partial \mathbf{v}}{\partial t} = \mathbf{D}\nabla^2\mathbf{v} + \mathbf{f}(\mathbf{v}) + \kappa, \qquad (1)$$

where $\mathbf{v}(\mathbf{x}, t)$, $\mathbf{x} \in \Re^2$, is the vector which represents the spatio-temporal distribution of excitatory and inhibitory macrostates, $\mathbf{f}(\mathbf{v})$ is the reaction term which describes the dipole cell interactions, the vector function $\kappa(\mathbf{x}, t)$ is the external stimulus which modulates the interactions between neural populations. The diffusion term is given by the product of the diffusion matrix $\mathbf{D}$ with the Laplacian on $\mathbf{v}$.

This system of coupled nonlinear partial differential equations (PDE) can be viewed as an assembly of a large number of identical local systems which are coupled together by a diffusive interaction. The distribution of excitation and inhibition $\mathbf{v}(\mathbf{x}, t)$ elicited by a stimulus, and spread by diffusion, can be interpreted as the *point image* [14] for a stimulus at a fixed position $\mathbf{x}'$. Given the homogeneity of diffusion processes over a local neighborhood, the point image may be conversely identified with the receptive field function. Hence, the diffusion component allows for a weighted averaging on the input signal κ over a region of a certain extent (the receptive field), thus extracting features from the image, whereas the nonlinear reaction components provide a flexible medium on which to base cooperative computation.

In this paper, we introduce a further interaction process formulated as integral terms, responsible for lateral recurrent inhibition within the same population. Hence Equation 1 can be explicitely rewritten as:

$$\begin{aligned} \frac{\partial v_E}{\partial t} &= D_E \nabla^2 v_E - \mathcal{A} v_E + f_E(v_E, v_I) + \kappa \\ \frac{\partial v_I}{\partial t} &= D_I \nabla^2 v_I - \mathcal{B} v_I + f_I(v_E, v_I), \end{aligned} \qquad (2)$$

with

$$(\mathcal{A} v_E)(\mathbf{x}) = \iint k_E(\mathbf{x}; \mathbf{x}') v_E(\mathbf{x}')\ d\mathbf{x}'$$

$$(\mathcal{B} v_I)(\mathbf{x}) = \iint k_I(\mathbf{x}; \mathbf{x}') v_I(\mathbf{x}')\ d\mathbf{x}'$$

$$f_E(v_E, v_I) = -a\ v_E - b\ g(v_I)$$

$$f_I(v_E, v_I) = -c\ v_I + d\ g(v_E)$$

where $g(\cdot)$ is a monotonically increasing function of sigmoid form; a, b, c, d are positive constants; k_E and k_I are the kernels of the corresponding integral operators $\mathcal{A}$ and $\mathcal{B}$.

The effect of the integral terms is strictly related to the shape of the kernels. In this work, particular attention has been devoted to those kernels that activate a recurrent inhibitory reaction through lateral clustered coupling, see Figure 1a. Such terms lead to oscillations in space and, at the same time, gate the temporal evolution of the system by supplying delayed negative influence from the neighborhood.

2.2 Coupled diffusion maps

Cortical cells of mammalian visual cortex are functionally specialized to respond to oriented local stimuli in the visual field [10]. The best patterns for such cells are typically not diffuse levels of illumination, but elongated bars or gratings oriented at specific angles. These orientation-selective cells have a highly regular organization on the cortical surface. Specifically, the distribution of orientation-selective cells reveals that the preferred orientation changes on the cortical surface in a regular manner. The orientation map $\theta(\mathbf{x})$ specifies for every point $\mathbf{x}$ the preferred orientation θ, ranging from 0 to π [2].

To model oriented receptive fields we can introduce anisotropy in the diffusion process. To model the organization of orientation maps we can allow space-variance in anisotropic diffusion coefficients. Under these hypothesis, the equations governing the evolution of neural macrostates is the following:

$$\begin{aligned} \frac{\partial v_E}{\partial t} &= \nabla \cdot \mathbf{D}_E \nabla v_E - \mathcal{A} v_E + f_E(v_E, v_I) + \kappa \\ \frac{\partial v_I}{\partial t} &= \nabla \cdot \mathbf{D}_I \nabla v_I - \mathcal{B} v_I + f_I(v_E, v_I), \end{aligned} \tag{3}$$

where now

$$\mathbf{D}_E = \begin{bmatrix} D_{11}^E & D_{12}^E \\ D_{21}^E & D_{22}^E \end{bmatrix} ; \; \mathbf{D}_I = \begin{bmatrix} D_{11}^I & D_{12}^I \\ D_{21}^I & D_{22}^I \end{bmatrix}$$

$$D_{11} = [D_x \cos^2 \theta(\mathbf{x}) + D_y \sin^2 \theta(\mathbf{x})]$$
$$D_{22} = [D_x \sin^2 \theta(\mathbf{x}) + D_y \cos^2 \theta(\mathbf{x})]$$
$$D_{12} = D_{21} = (D_x - D_y) \sin \theta(\mathbf{x}) \cos \theta(\mathbf{x})$$

with D_x and D_y the principal diffusion coefficients. The processes modeled by these equations can be considered as *coupled diffusion maps.*

If we consider orientation maps slowly varying over distances comparable to the extent of a point image, the distribution of orientations can be assumed almost uniform, except for local discontinuities. Thus, the point image can be still related to the receptive field function. Moreover, by assuming the global non isotropicity of the diffusion processes we obtain point images elongated along preferred directions, leading to continuous distributions of oriented receptive fields similar to biological orientation maps $\theta(\mathbf{x})$ [10] [4] [2].

As it has been anticipated in the previous section, the geometrical organization of inhibitory recurrent couplings is fundamental for defining the local spatial behavior of the system. In fact, it has been shown in a previous work [3] that regular spatial alternation of antagonist activations arises spontaneously as an effect of recurrent inhibitory couplings among clustered populations of cells, whereas no oscillations have been observed when inhibitory connections are uniformly distributed in the horizontal plane. Thus, the kernels k_E and k_I used in the model present meaningful values only at a certain distance from their centers, and along a direction orthogonal to the principal axis of diffusion (i.e. orthogonal to the receptive field orientation), see Figures 1b and 1c. For

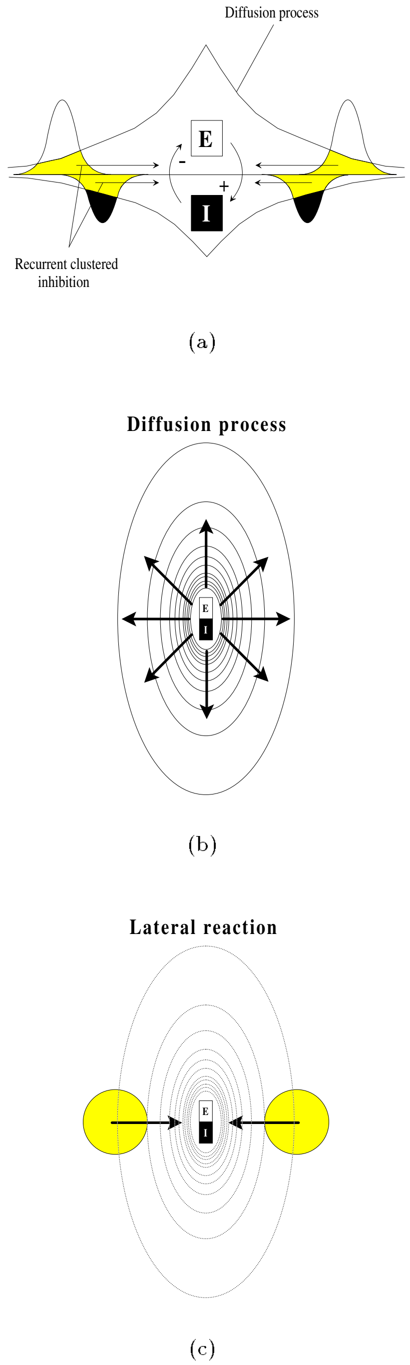


Figure 1: (a) Schematic representation of the reaction processes that take place in each location of the map: local competition within the dipole cell and laterally-displaced inhibition. The white and black Gaussians represent the kernels k_E and k_I respectively; (b) the 2D spatial organizazion of diffusion and reaction (c), assuming that the receptive field is oriented along the dipole axis. The shaded areas represent the net contribution to recurrent inhibition from a neighborhood of the dipole cell.

these kernels, $\mathbf{v}(\mathbf{x}, t)$ presents oscillations in space and cortical point images (i.e. receptive fields) show a characteristic Gabor-like shape, similar to the profiles of real cortical cells [11] and useful as "economic" space/frequency image decriptors [5]. It is worthy to note that, since our model is based on average properties of a given cortical area, it cannot represent the individual behavior of single cells. It can only indicate properties probably common to all or most neurons within a given area (i.e. macrostates).

Under appropriate input conditions, local competitive terms ignite temporal oscillations similar to the ones experimentally observed in the local field potentials of visual cortex [7] [13] [9] (see Figure 2). In such a way, oriented spatio-temporal fields characterize the stimulus-response coupling via a neural mapping $\kappa \rightarrow \mathbf{v}$ which specifies the local visual cortical representation of the image.

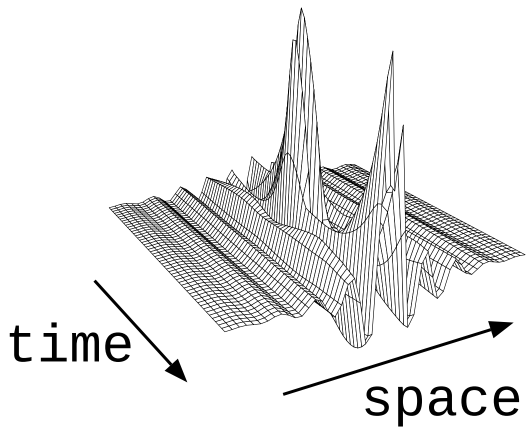


Figure 2: Spatio-temporal representation of a modeled local field potential.

To bind different features in a global unifying percept, we need to establish interconnections among the temporal phases of oscillations. Such interactions encode global information about the position and orientation of the input stimulus. To this purpose, superimposed weak coupling between localized portions of the orientation map impose a long-range coherence and the whole system evolves through different configurations of coherent states. These weak interactions do not affect receptive field dimension and shape. In the following examples we choose the distribution of diffusion coefficients according to a "patch-like" map as the one depicted in Figure 3. Sparse long-range connections link the centers of neighboring orientation domains in accordance to the scheme depicted in the figure.

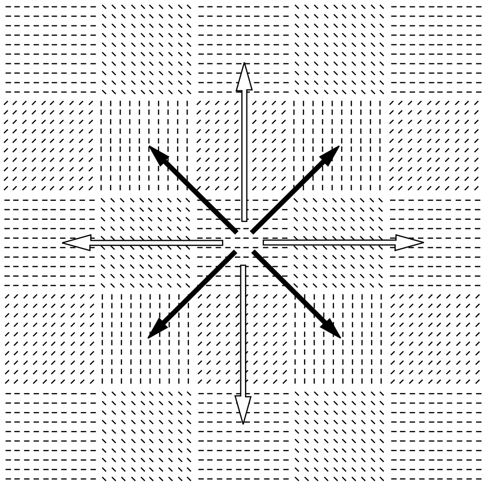


Figure 3: Example of orientation map. The long-range couplings between neighboring orientation domains are evidenced. Black arrows indicate an inhibitory influence whereas white arrows indicate an excitatory coupling.

Connections are allowed to be either positive or negative, according that they link (respectively) iso-orientation domains, or domains that are characterized by orthogonal orientation properties. The approach can be straightforwardly generalized to more realistically looking maps [4]. Further improvements to the model can be achieved by substituting fixed long-rang connections with dynamic hebbian and anti-hebbian linkings similar to the one proposed by [18] [17].

3 Experimental results

Coupled diffusion maps are able to detect texture differences in an image on the basis of different amplitude and phase responses of cells selective to different orientations. For testing network performances we have considered both natural and artificial texture images containing different oriented textures. The input image activates a discrete set of interaction sites (pixel grid).

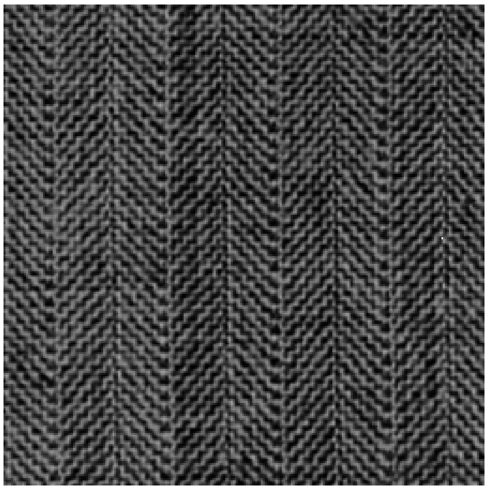
Simulations are performed by feeding it to the *coupled diffusion map*, and by looking at the distribution of excitation on the map, that has been previously initialized with a small level of noise to break symmetries. The system evolves and different texture structures result in different patterns of excitation on the cortical map, according that the orientation of the receptive fields matches the orientations of textural elements. If the test image is composed of repeated oriented elements, locations in the regions of the map selective to that orientation have the strongest output. After time evolution, the system reaches an equilibrium state in which regions with uniform texture lead to similar dynamic patterns of activation. Specifically, the regions of the map encoding orthogonal textures converge to phase-opponent equilibrium states, while regions encoding similar textures are phase-locked. Averaging the pattern of activity over a window comparable in size with those of a complete set of orientations (hypercolumn) we gain instantaneous global perception supporting scene segmentation. In such a way, we gain a global percept without losing individual response properties of single cells.

An example of segmentation performance on natural texture image is shown in Figure 4. Ignoring the border effects, regions with uniform texture structure are well segregated.

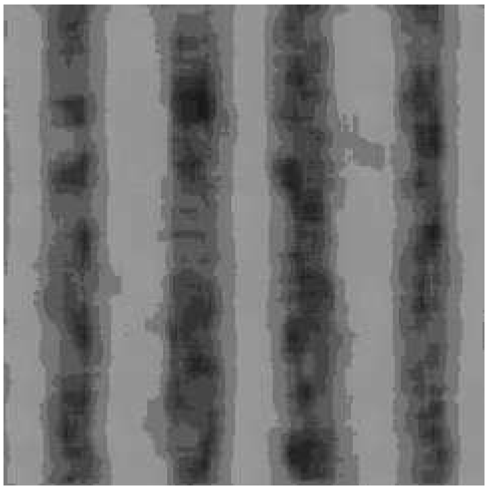
4 Conclusion

In this paper, we have proposed a model of cooperative computation for texture image segmentation. In this approach the featural elements present in the image are combined, through reaction-and-diffusion paradigm. Typically, AI solves these grouping processes evoking what the system "knows" about the image/object. On the contrary, the emphasis here is on the exploitation of grouping mechanisms based on cortical-style computation that is applicable without recourse to specific *a priori* knowledge. Visual cortex is modeled as a *coupled diffusion map*. Each location *maps* different featural elements, thus realizing a reduction in the complexity of the image, and the whole system gains relational descriptions through temporal synchronization. Simulations show that the average characteristics of the local-field potentials aggregates local information of small assemblies of cells selective to a complete set of orientations. The spatio-temporal characteristics of such local-field potentials yields global perception about edges separating one region from another and determine regions on the basis of continuity of structure. In this way, general principles of grouping processes such as *proximity,similarity*, and *continuity of structure* are transferred in architectural solutions that allows an intelligent assignment of computational/architectural resources.

It should be observed that the architectural model of *coupled diffusion maps* can substantially enrich the computational functionality of active resistor meshes, recently proposed as an efficient implementation of low- and intermediate-level image processing [15].



(a)



(b)

Figure 4: (a) A natural image presenting textural elements oriented along 45 and 135 degrees; (b) a snapshot of the averaged activation of the coupled diffusion map: orthogonal textures reach coherent phase-opponent equilibrium states.

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