Data Driven Time–Frequency Decompositions for Information Fusion

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Outline

- Concept of Data Fusion Via Fission
- Empirical Mode Decomposition (EMD)
- Hilbert-Huang reconstruction
- Some Illustrative Examples
- Multidimensional EMD
- Application 1: Image Restoration
- Application 2: Heterogeneous Image Fusion
- Application 3: Out of Focus Image Fusion
- Extensions to 3D
- Conclusions
About Imperial College
A General Data Fusion Problem

How do we make a mental ”image” of a meal

- **taste** bitter, rotten
- **smell** pleasant, spices
- **vision** colour, presentation, rare, medium, well done
- **touch** bread, toast
- **hearing, temperature, toughness**
Signal processing algorithms for “sensor” or “data fusion” are based on:

- **Probabilistic models**: Bayesian reasoning, belief propagation, evidence theory, robust statistics;

- **Least squares**: Kalman filtering, regularization, set membership;

- **Intelligent fusion**: Fuzzy logic, neural networks, genetic algorithms;

- **Time–frequency analysis based fusion**.
Benefits of the Data Fusion Approach

The synergy of information fragments offers some advantages over standard algorithms, such as:-

- Improved confidence due to complementary and redundant information;
- Robustness and reliability in adverse conditions (smoke, noise, occlusion);
- Increased coverage in space and time; dimensionality of the data space;
- Better discrimination between hypotheses due to more complete information;
- System being operational even if one or several sensors are malfunctioning;
- Possible solution to the vast amount available information.
Some Literature
Data Fusion via Fission

- Fission: Decomposition into "particles"
- Fusion: Recombination of particles into the desired signal
Empirical Mode Decomposition

The empirical mode decomposition (EMD) is a fully data driven method for decomposing (multicomponent) signals into a set of modes that are:

- monocomponent
- naturally derived basis functions

The signal, $x$, is decomposed into the following

$$x = \sum_{i=1}^{n} c_i + r$$

where $c_i$ denotes the $i$-th IMF, $i = 1, \ldots, n$ and $r$ the trend

⇒ No prior assumptions are made about the signal, ideal for nonlinear and nonstationary analysis
Intrinsic Mode Functions (IMF)

An IMF a function that satisfies the following two criteria:

1. The upper and lower envelopes are symmetric

2. The number of zero-crossings and the number of extrema are equal or differ at most by one

IMF Properties

- Orthogonality
- Amplitude/Frequency modulated components
- “Oscillatory modes” that facilitate time-frequency analysis
Empirical Mode Decomposition: Underlying Idea

- Creates an adaptive decomposition of the signal in hand.

- The basic idea behind EMD is to consider an input signal as fast oscillations superimposed on slow oscillations.

- The fast oscillations are repeatedly sifted from the input signal until a monotonic signal (residue) is obtained.
Sifting Process - IMF Decomposition

1. All the maxima of the signal are connected with a spline denoted by $U$

2. All the minima of the signal are connected with a spline denoted by $L$

3. Determine the mean envelope $m = \frac{U + L}{2}$

4. Subtract the mean envelope from the signal to obtain a proto-IMF
Sifting Process - IMF Decomposition

5. The process is repeated until the sifted signal satisfies the conditions of an IMF.
6. After extracting an IMF, this same IMF is subtracted from the signal and the residual is treated as the new data and fed to step 1.

7. All above steps are iterated until the final residual is a monotonic function. The last residual is considered as the trend.
**Instantaneous Frequency**

The IMFs are *narrowband* signals ⇒ the Hilbert transform can be applied to each IMF separately.

IMFs can be represented as a set of analytic signals

\[ c_i(t) = a_i(t) \cdot e^{j\cdot\theta_i(t)} \]

The whole analytic signal thus is

\[ X(t) = \sum_{i=1}^{n} a_i(t) \cdot e^{j\cdot\theta_i(t)} \]

This process can be described as “fission” of a multicomponent signal.
Instantaneous Frequency

Defining the instantaneous frequency as the derivative of the phase

\[ f(t) = \frac{d\theta}{dt} \]

The Hilbert Spectrum can be obtained by contouring the instantaneous amplitude \( a(t) \) versus time \( t \) and instantaneous frequency \( f(t) \)

⇒ We can calculate the instantaneous frequency of each IMF component at each time instant.

😊
Some Examples

Sum of two sinewaves

Time frequency representation
Illustrative Example

Hilbert–Huang spectrum (right) has better localisation properties than wavelets (middle)

Image Enhancement and Fusion Using EMD

- Image features (object texture or unwanted noise) can be attributed to local variations in spatial frequencies.

- Therefore, the behaviour of the extracted image modes can reflect these features.

- Correct fusion of the “relevant” IMFs can be used to highlight (or remove) specific image attributes.

We consider the fusion capabilities of EMD under the following headings:

- Image Denoising

- Image Restoration (Illumination Removal)

- Image Fusion (of Images From Multiple Image Modalities)
Image Denoising

Noise contamination is a common problem when acquiring real world images and consequently image denoising is an important element of image processing. Many existing methods, however, are sub-optimal for the following reasons:

- They make unrealistic assumptions about the data (ICA - unrealistic independence conditions, PCA - noise and original image can be separated by linear projection)

- They are not optimised for enhancing higher order (nonlinear) statistics, that are commonly associated with the perceptual quality of an image, and do not cater for other real world data characteristics such as nonstationarity (block based Weiner filtering)

- They are computationally complex (Bayesian and particle models)
Image Denoising

Consider an original image corrupted by white Gaussian noise.
Decomposing the contaminated image by EMD, we obtain the following:

Note how each of ‘Image Modes’ represents the frequency scales within the image. The higher index IMFs contain high frequency detail such as the image edges while slowly oscillating effects such as illumination are contained within the low index IMFs.
Roles of IMFs

Clearly, we can operate at different scales
Automated Data Fusion via Fission (MLEMD)

EMD for Data Fusion via Fission

Incorporating the scale and temporal information
Image Denoising (PREMD)

We can deduce that the ‘most’ of the noise energy is contained within the high frequency modes. One approach, is to empirically select the IMFs that best represent the original image.

- The new SNR is 15.2 dB, the noise energy is clearly reduced
- The approach is clearly suboptimal (binary weighting of the IMFs, not based on any optimality criterion)
Another approach is to determine the best estimate of the original signal by combining the IMFs in the least mean squares sense.

- The new SNR is 16.7 dB, an improvement over PREMD
- The approach is block based and static, and cannot cater for dynamic IMF relevance estimation
A more adaptive and robust solution is to combine EMD with machine learning (adaptive filtering)

- High quantitative performance, the SNR is 18.7 dB.
- The approach is dynamic and facilitates local feature fusion
- High qualitative performance, the perceptual qualities of the original image are retained
Image Denoising

Noisy image, SNR = 13 dB

EMD of the image

Denoising using PREMD, SNR=17.5 dB

Denoising using MLEMD, SNR=21 dB
The Method Naturally Deals With Texture

Cracked varnish on wood: Left – original; Midle – cracks; Right – wood pattern

Carpet: Left – original; Midle – texture; Right – carpet pattern

The texture is separated naturally as higher frequency T-F components
A key problem for a machine vision system is image changes that occur due to scene illumination.

Incident light on a surface produces complex artifacts, making it difficult for the system to separate changes caused by local variations in illumination intensity and colour.

- It can be assumed that shade in images creates low valued regions with large extrema that change slowly.
- It is therefore likely that the effects of the shade will be isolated in the lower index IMFs and a shade free image can be achieved by combining the relevant IMFs.
Illumination Removal – Real World Objects

Shade removal: the shading is now uniform across the image surface
Image Restoration (Illumination Removal)

Image with shade  Shade only  Original image

Shade removal: Also works on real-world objects
Image Fusion (From Multiple Image Modalities)

- Image fusion is becoming an important area of research, particularly as different methods of image acquisition become available.

- The fused image retains all “relevant information” from the different sources while disregarding unwanted artifacts.

- Given the unique “fission” properties of EMD, it has a strong potential for fusion.

- We propose the use of complex EMD with the input images as real and imaginary components respectively.

- The instantaneous amplitude of the extracted IMFs indicates, for each frequency level at each pixel, which of the components contains the salient information. Fusion can be achieved by combining only IMF components with the largest instantaneous amplitudes.
Obstacles to Automatic Heterogeneous EMD Fusion

- However, the fully adaptive and empirical nature of the algorithm, as well as a sensitivity to parameter selection, compromises the uniqueness of the decomposition.

- Therefore signals with similar statistics can often yield different IMFs (in both number and frequency) making it difficult to compare decompositions from different sources.

This is illustrated by observing the decompositions of a sinusoid corrupted by different realisations of AWGN. Note the difference in the IMFs.

7 IMFs 8 IMFs
Obstacles to Automatic Heterogeneous EMD Fusion

Automatic fusion algorithms are necessary for widespread use!

But this is not often possible using standard EMD because

- Uniqueness of the scales cannot be guaranteed;
- Comparison of IMFs from different sources is meaningless!

Thus, automatic fusion of heterogeneous sources using EMD is only possible if their IMFs are

- equal in number;
- matched in properties (frequency).

[Mandic et al., Flandrin et al.]
Complex EMD - Local Mean Estimation

(a) RIEMD

(b) BEMD

(c) A complex wind signal

(d) IMF6 + IMF7
Heterogeneous EMD Fusion

- It was proposed [Looney and Mandic] to use the complex extensions of the algorithm to decompose heterogeneous sources simultaneously.

- The approach may be used to find “common scales” within different data sets, thus addressing the problem of uniqueness.

Observe how common frequency scales are found in different signals (U1 and U2) by applying complex extensions of EMD to (U1 + jU2).
Fusion Results [Looney and Mandic ICDSC’08)

Visual 
Thermal 
Pixel Average Fusion 
PCA Fusion 
Wavelet Fusion 
Complex EMD Fusion
Out of focus image fusion using complex EMD

Complex EMD vs Wavelets

EMD fusion  Wavelet fusion

The wavelets produce artifacts - around the text visible as shaded “boxes”
Other Possibilities – “Environmental Dimension”


Here, the fusion was performed manually, without using any machine learning or extensions of EMD.
Conclusions - Image Fusion

□ EMD is non-parametric and self adaptive which is advantageous when decomposing real world images, which display nonstationary and nonlinear behaviour, into their natural frequency modes

□ It is a powerful tool for the purposes of “image fusion via fission”

□ Automatic algorithms for enhancement, restoration and fusion have been presented

□ This is all achieved within a unique framework

□ Extensions include direct 2D and 3D realisations, which also facilitates the processing of color images